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Draining of a Fluid from a Rotating Cylindrical Tank

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Introduction

PREVIOUS investigations have shown that in draining a low-viscosity liquid from a stationary tank, there exists a fluid level at which a dip on the free surface extends almost instantaneously into the drain.[1–5] The height at which this occurs, referred to as the critical height,[5] has been shown to be a function of the ratio of inertial forces to body forces, a Froude number, here defined as $Fr = \bar{U}^2/gR$, where $\bar{U}$ is the average free-surface velocity, g the acceleration of gravity, and R the tank radius. However, there are no data on a similar occurrence when the tank is rotated with constant angular velocity ("rigid body" rotation). It is the purpose of this Note to present quantitative data and qualitative observations on the occurrence of a critical height in a uniformly rotated flow. These data and observations should lend insight into the nature of the flow, so that a mathematical model of the problem can be constructed.

The problem investigated is shown in Fig. 1. A low-viscosity fluid (i.e., water) in a closed and continually pressurized cylindrical tank, filled at the centerline to an initial height H_i, is rotated uniformly about its central axis. At time = 0, flow is initiated through a symmetrically placed drain of radius $a(a/R \ll 1)$. At some later time > 0, rotation other than rigid body can be detected. As the fluid height decreases, the deviation from uniform rotation increases; the free surface is no longer parabolic but has a depression at the center. The free surface eventually reaches a critical height, H_c, at which the depression extends to the drain almost instantaneously and a vortex is formed.

As explained by Morton,[6] in a flow with externally imposed rotation, the effects of rotation are controlled by the Rossby number (ratio of inertial to Coriolis forces). In the present case, the Rossby number is defined as $Ro = \bar{U}/\Omega R$

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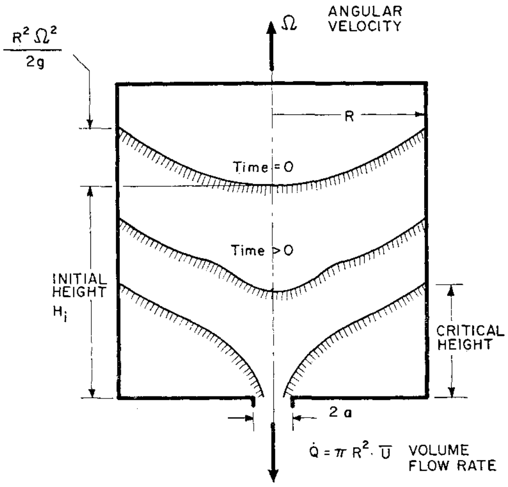

Fig. 1 Free-surface profiles.

where Ω is the rate of angular rotation of the tank. Various investigators have shown that two distinct flow patterns can occur in a uniformly rotated flow, depending on the value of Rossby number. Of these investigations, those of Stewartson,[7] Barua[8] and Long[9] are applicable to the present study.

Stewartson[7] and Barua[8] both investigated the streamline pattern obtained at values of $Ro \ll 1$ for a point sink in a uniformly rotating flow. They showed that the flow was directed toward the sink as a cylindrical jet surrounded by a uniformly rotating flow. Long[9] obtained a mathematical solution for sink flow from a rotating cylinder of infinite length. The validity of the solution was limited to values of $Ro \geq 0.522$. The solution showed that as Ro decreased from infinity (no rotation) to this limit, the sink drew an ever-increasing amount of flow from a region near the axis of rotation. Long hypothesized that for values of Ro below this limit, the flow might be unsteady and inertial waves could exist in the flow. This limiting value of Ro is of the same order of magnitude as that discussed by Greenspan,[10] ($Ro \simeq 2/\pi$), below which rotating flow could exhibit columnar motion.

Thus, H_c should be a function of Ro as well as Fr, and qualitative differences in the flow should be apparent at $Ro \sim 0.5$. However, whereas in the nonrotating case ($Ro \to \infty$, $1/Ro \to 0$) H_c was independent of H_i, at values of $Ro < 0.5$ the hypothesized unsteady nature of the flow should indicate some dependence, on H_i.

Apparatus

The experimental setup is shown schematically in Fig. 2. To calculate the three dimensionless groups H_c/R, Fr and $1/Ro$, H_c, $\bar{U}$ and Ω must be measured. A turbine flow meter was used to measure the volume flow $Q = \pi R^2 \cdot (\bar{H}_i - \bar{H})$, where $\bar{H}_i$ and $\bar{H}$ are average values, and the volume rate of flow $\dot{Q} = \pi R^2 \cdot \bar{U}$, both as functions of time. The value of $\bar{H}c$ ($\simeq Hc$ for the values $1/Ro$ and Fr obtained in the experiments) was determined by a visual sighting of the point of free-surface extension, and by electrically interrupting the flow meter signal at this point. Measurement of Ω was accomplished by an electronic counter activated every tank revolution by a cam-microswitch arrangement on the tank turntable.

The test procedure was as follows: First, the tank was filled to the desired $\bar{H}_i$ and kept stationary for a time to damp out any residual vorticity. Then rotation was started. Spinup times were calculated from the analysis of Wedemeyer.[11] The applicability of his results was verified by a series of tests to determine the time it took a ½-in. diam paddle-wheel float, supported at the centerline and starting from rest, to attain the angular velocity of the tank. Upon completion of spinup the tank was pressurized. The pressure was held constant during draining, and the critical height was recorded.

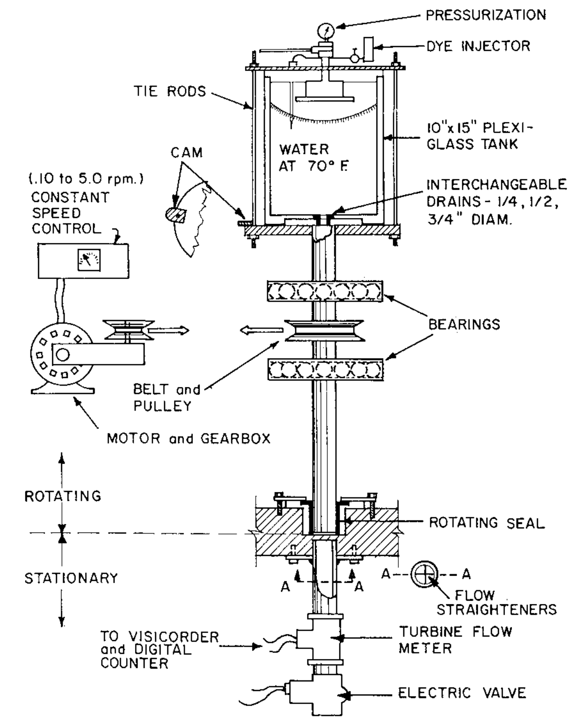

Fig. 2 Experimental apparatus.

Results and Discussion

Measurements were obtained for H_c/R, Fr and $1/Ro$ over the following range and values of uncertainty: $0.69 \times 10^{-4} \pm 7\% \leq Fr \leq 2.65 \times 10^{-4} \pm 1\%$, $0.10 \pm 2\% \leq 1/Ro \leq 5 \pm 1\%$, $0.10 \pm 15\% \leq H_c/R \leq 5 \pm 1\%$. The results of the experiments are summarized in Fig. 3. The following observations can be made with regard to the data. 1) The results for no rotation ($1/Ro = 0$) agree with the predictions of Lubin and Springer.[5] 2) For values of $1/Ro \lesssim 0.10$, the effect of rotation on the critical height is negligible. 3) For $1.0 \lesssim 1/Ro \lesssim .10$, H_c is independent of H_i. In this region a dye trace (Fig. 4a) describes a streamline, the flow being quasi-steady, and has no perceptable relative rotation except near the drain. This pattern maintains itself up to the critical point. It would appear that for values of $Ro > 1.0$, the flow is primarily controlled by the Coriolis force due to the angular rotation and the radial component of the sink flow near the drain. This effect appears to be confined to a region near the drain. 4) For values $1/Ro > 1.0$, H_c is a strong function of both Ro and H_i. The value of this limit is of the same order of magnitude as that predicted by Long[9] for the point at which sink flow in an

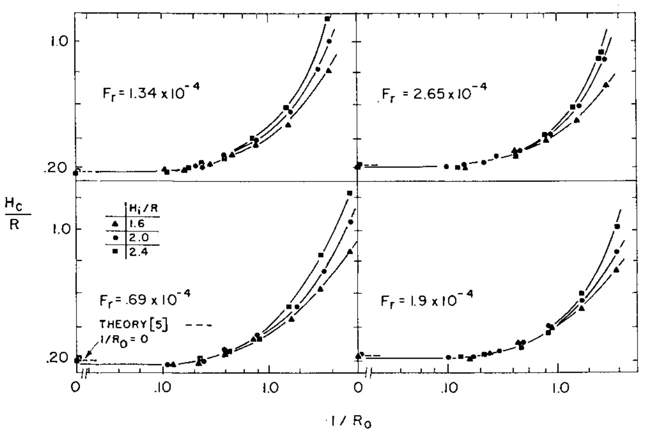

Fig. 3 Experimental data.

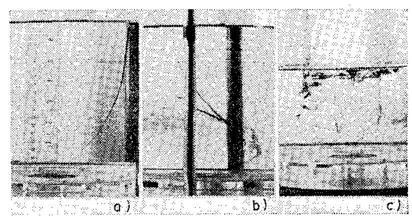

Fig. 4 Dye traces. a) $1/R_0 = 0.425$. Dye trace in sink region. b) $1/R_0 = 4$. Coil pattern with fluid height greater than critical value. c) $1/R_0 = 4$. Ink wall pattern with fluid height less than critical value.

infinite rotating cylinder can have inertial waves that propagate upstream at velocities greater than the free surface velocity.

According to Long,[9] the wave with the fastest velocity is that with no internal nodes, i.e., with wavelength equal to H_i. Thus, the effect of the Coriolis force-produced relative rotation near the drain is propagated upstream to the free surface faster at larger values of H_i. Consequently, the data show an increase in H_c with H_i at constant values of Fr and $1/Ro$.

This would tend to explain that, while $\bar{U}$ is quasi-steady, the flow within the fluid is not. The dye trace, which now describes a streak line, has the following time history. It proceeds to the drain with no relative rotation except near the drain itself. This slight swirl pattern then proceeeds to wind up into a helical pattern as the fluid attains a strong relative rotation away from the drain (Fig. 4b). This coil pattern rotates even faster, until the dip characteristic of the critical height is observable on the free surface. The dip starts to accelerate towards the drain, at which time the dye trace has formed into a series of concentric cylindrical surfaces (Taylor "Ink Walls",[12] Fig. 4c). The flow now has the columnar pattern usually associated with strongly rotated flows in which the Coriolis and centrifugal forces dominate the other inertial forces.

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